Market design with centralised wind power management: handling low-predictability in intraday markets

Arthur HENRIOT[1]

Abstract:

This paper evaluates the benefits for an agent managing the wind power production within a given power system to trade in the intraday electricity markets, in a context of massive penetration of intermittent renewables. Using a simple analytical model we find out that there are situations when it will be costly for this agent to adjust its positions in intraday markets. A first key factor is of course the technical flexibility of the power system: if highly flexible units provide energy at very low prices in real-time there is no point in participating into intraday markets. Besides, we identify the way wind production forecast errors evolve constitutes another essential, although less obvious, key-factor. Both the value of the standard error and the correlation between forecasts errors at different gate closures will determine the strategy of the wind power manager. Policy implications of our results are the following: low liquidity in intraday markets will be unavoidable for given sets of technical parameters, it will also be inefficient in some cases to set discrete auctions in intraday markets, and compelling players to adjust their position in intraday markets will then generate additional costs.

Keywords: Market design, intraday markets, wind forecasts, large-scale renewables, intermittency

---

[1] Research assistant at Florence School of Regulation, Villa La Fonte - Via delle Fontanelle, 10 – 50014 San Domenico di Fiesole (FI) –Italy. Email: arthur.henriot@eui.eu

The author would like to thank Jean-Michel Glachant, Vincent Rious, Haikel Khalfallah, Marcelo Saguan, the editor and three anonymous referees for their highly capable help. Valuable comments were also provided during the YEEES seminar that took place in spring 2012. All errors remain of course the author's own responsibility.

1. Introduction

The integration of a significant share of variable renewables in the electricity generation mix is a source of economic and technical challenges. Wind generation variability and low-predictability constitute a major obstacle to the integration of wind farms into electricity markets[2].

We study in this article the case when intermittent RES are not isolated form electricity markets and considered as a standard generator and we focus on one of the possible solutions to manage the low predictability of electricity generation by wind farms: the use of Intraday Markets (IM). Wind forecasts improve significantly when realised closer to generation. Giving generators a chance to adjust in the IM their commitments realised in the Day-Ahead markets could help renewables to lower their imbalance costs.

Intraday markets give players an opportunity to trade and to modify their production schedules after the day-ahead gate-closure. They are already in place in most European countries but their design is subject to significant variations. They can in particular be continuous (Germany, Denmark, France) or feature discrete auctions (Spain, Italy). Despite wind already representing a significant share of generated electricity in several countries, liquidity in IM remains low and the share of electricity traded in IM is quite incidental[3]. Complementary rules have sometimes been put into place to increase liquidity. For instance, from January 1, 2010, TSOs are required in Germany to balance any difference between volumes of power from renewable sources sold in the Day-Ahead auction and the feed-in based on the intraday forecast (Besnier, 2009). While such a regulatory measure will lead to a higher liquidity in the IM, we argue it could also lead to additional costs. The purpose of this article is to study under what conditions it will be beneficial for wind generators to trade in IM to manage wind low-predictability.

[2] For more details, the reader can for instance refer to the recent study by MIT: Perez-Arriaga, I.J., 2012. Managing large scale penetration of intermittent renewables. MIT.

[3] In 2009, the volume traded within the organised IM in Germany was 4.2% of the volume traded in the organised Day-Ahead market. For the same year in Spain, the volume traded in the MIBEL IM was under 16% of the total volume traded within the organised markets. (Source: Barquin et al. 2011)

We build a simple analytical model to study how the prediction error for electricity generated by wind farms for a given generation time can be managed in IM. In order to focus on the effects of low-predictability for a single hour, we do not consider interdependency between adjacent generation times. We suppose wind generators are aggregated into a single player who commits to generate a given quantity in Day-Ahead markets. Due to forecast errors, this player is exposed to imbalance costs when the actual output is different from its financial position. This player is also given the possibility to adjust its commitments by interacting with thermal generators[4] at a set of gates within the IM. We also introduce a parameter to take into account the system flexibility in our model. Due to the limited technical flexibility of thermal generators, it is more expensive to procure energy on short-notice[5].

We use this model to study the average profits of a wind power producer using the best predictions available to adjust its position in selected gates from IM and compare it to the average profits realised by a producer adopting a more passive attitude. It is less expensive to manage imbalances earlier, but there is a risk of correcting self-compensating deviations. This process allows us to establish a set of critical values for the technical properties of the forecast error. Relevant parameters include standard error, correlation between errors at different times, and additional costs of purchasing electricity closer to real-time. Our results indicate that the value of these parameters will determine whether it is a good strategy for the producer to use updated predictions to trade in the intraday market at a given time. As these parameters evolve with each gate-closure time, setting discrete auctions at a sub-optimal time will deter participants from trading within this time period.

2. Previous works

Despite the relatively low volume of electricity currently traded in intraday markets, their alleged potential to assist the integration of intermittent renewables such as wind led to the development of a range of studies focusing on this topic. For example, Borggrefe and Neuhoff (2011) and Hiroux and

[4] In this article, the term “thermal generators” is used to refer to all units considered as having a predictable output. Thus large hydropower can also be included in this abusive simplification.

[5] According to a recent study by the MIT Energy Initiative (2012), nuclear plants (featuring low marginal costs) require six to eight hours to ramp up to full load, while coal plants can ramp their output at 1.5%-3% per minute. The most flexible coal units are the smaller and older plants with less efficiency.

Saguan (2010) both mentioned the use of IM to manage wind low-predictability. Borggrefe and Neuhoff (2011) presented intraday markets as a tool to keep the volume of balancing services low in systems featuring a significant penetration of intermittent renewables but did not consider oscillating predictions[6]. Hiroux and Saguan (2010) argued setting the gate closure that closes intraday markets near real-time would help to reduce wind integration balancing costs.

A first category of studies focusing on the IM consists of empirical analysis of players' participation in IM, such as Weber (2010) and Furió et al. (2009). Weber (2010) focused on the volume exchanged in several European Intraday Markets. He estimated a theoretical potential for position adjustments of wind generators in intraday markets and deduced that the amount of exchanges reached in these markets were quite low when compared to this potential. Weber (2010) distinguished two possible explanations for poor liquidity. A first reason could be poor *market design*. In this case, it can moreover become a self-sustaining phenomenon, as the absence of liquidity reduces the trust of participants into IM. Another possible explanation can be the absence of a real need for IM, i.e. a question of *market structure*[7]. There is a fundamental difference between these two drivers with consequences regarding policies to adopt. Yet no clear conclusion was reached regarding the exact source of low liquidity. Furió et al. (2009) realised a statistical analysis of trades made in the Spanish Intraday markets. This study revealed that about two thirds of the exchanges realised within each of the six trading sessions were linked to the hourly horizons negotiated for the last time in this session. Most of the time only one gate out of six was really used by participants. They furthermore added the low liquidity calculated could be due to an absence of need to make adjustments in the IM.

A second category of studies features models to estimate the value for wind power generators of trading into intraday markets. Usaola and Angarita (2007) considered three possible strategies in IM: no bidding, bidding best prediction, and an "optimal" strategic bidding. The frame was the Spanish IM, prices were inputs based on historical data, and only one intermediate step was considered in the

[6] The term "oscillating predictions" refers to the case when the successive updated forecasts for the same generation time are alternatively increasing and decreasing when getting closer to real-time.
[7] Stoft (2002) for instance employed "market structure" by opposition to "market architecture" to refer to properties of the market closely tied to technology and ownership. We will stick to this definition in this article.

IM. Results indicated bidding the best prediction was not the optimal strategy and that it was sometimes even preferable not to play at all in IM. Similar results were obtained by De Vos et al. (2011) in the Belgian context. Day-Ahead (DA) and Balancing Mechanism (BM) prices were inputs taken from the Belgium Power Exchange BELPEX while IM prices were estimated through linear interpolation between DA and BM prices. Increasing total balancing costs resulting from trading into IM were explained by oscillating predictions. Maupas (2008) employed a quite sophisticated approach using a power system simulation and modelling the interaction between intraday and balancing markets. He established that it was not beneficial to trade into IM with poor liquidity due to interactions between the different hourly provision horizons. In Maupas' model, poor liquidity was an exogenous input taken into consideration by setting intraday market prices closer to the BM prices than to the DA prices.

While wind and intraday markets have hence been subject to different approaches, we believe there is room for further investigation. While there seems to be a general intuition in the studies mentioned in this section that trading in IM could result in higher costs in case of poor liquidity and oscillating predictions, the calculations made so far did not establish for what kind of forecast precision and for what market flexibility it was the case. By using a simpler analytical model, we might not be able to deliver accurate numerical results but we will be able to focus on the role played by two key technical components: forecast accuracy and system flexibility.

3. Model

3.1. Modelling framework

In our analytical model, wind generators are aggregated into a single player. This player could represent a utility operating the totality of wind power plants, a national aggregator, or a TSO responsible for managing wind intermittency as it is the case in Germany. Our results can be applied to any system featuring one of these structures.

Our player generates energy using installed wind capacity W and is also able to procure energy from thermal generators in electricity markets. At the gate-closure time of the day-ahead market, this player plans to generate a given quantity of wind energy for a final production horizon. However due to imperfect forecast, the final output will be different from the player position. This "wind player" will therefore need to manage imbalances.

We compare different strategies in our model ranging from a *completely passive strategy* to an *extremely active strategy*. A *completely passive strategy* is to "do nothing" and pay the balancing costs when the final production is realised: this is the case when it is not possible to trade into IM or when the player is not taking part into these markets. An *extremely active strategy* is to use the updated forecast available at each gate of the IM: the wind player will then be interacting with the thermal generators to adjust its positions[8]. As a result, the active player will need to buy or sell less energy in balancing markets (only the remaining error at the last intraday market gate closure) but might buy and sell more energy in the intraday markets due to oscillating predictions. The completely passive strategy and the extremely active strategy constitute the two extreme possibilities of a much more complex set of strategies: in practice, in our model, at each available gate of the IM, the player can choose whether to adjust its position using the best available forecast. This is illustrated in Figure 1.

We assume that the evolution of the system imbalance is driven by the wind manager generation imbalances, which is a reasonable assumption in a system featuring a significant share of variable renewables managed by a single player[9]. Indeed while load is also uncertain the errors will then be smaller and their evolution is easier to anticipate (Maupas, 2008).

---

[8] If the updated forecast indicates a higher output than the previous forecast the wind player can sell more energy. If the updated wind forecast indicates a lower output the wind player must buy energy.

[9] The assumption that a single player is managing the whole wind power generation is therefore a key assumption in our discussion. Our results would however remain qualitatively true with a significantly dominant player or for any player whose imbalances are strongly positively correlated to the total system imbalances.

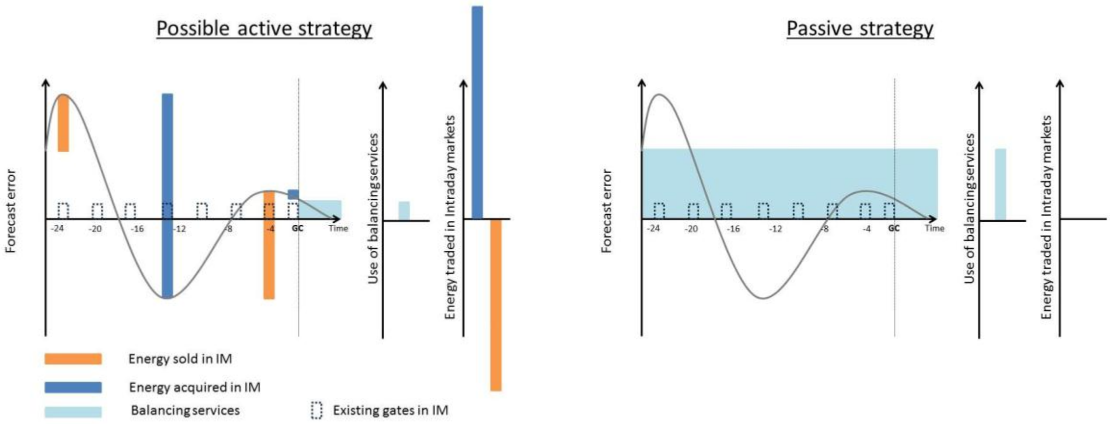

Figure 1: Illustration of two possible strategies: the player chooses to participate in IM at gates H-24, H-12, H-4 and H-2 (left side) vs. the player decides not to participate at all in IM (right side).

Thermal generators have a limited flexibility. The least flexible plants will not be able to adapt their production to the demand when getting closer to the production horizon or will only be able to adapt it in a restricted way respecting ramping constraints. They will therefore withdraw part of their offers from the supply function, as illustrated in Figure 2. The resulting inverse supply function will therefore feature a steeper slope, and prices will get more expensive when getting closer to the production horizon[10]. Moreover, the units most likely to provide the required flexibility to manage wind variability are usually the ones with high marginal costs[11] (see IEA (2012)).

At last, energy procured in real-time is not always charged at cost-reflective prices (Vandezande et al., 2010). Penalties can be imposed by the system operator to provide ex-ante balancing incentives to participants. Such penalties could be included in our model by higher prices for energy procured and lower revenues from selling energy in real-time markets. Due to these extra-costs, participants should then have higher incentives to participate in intraday markets.

[10] Exercise of market power could strengthen the impact of this phenomenon, as illustrated by Green and Vasilakos (2010): when the residual demand for power production by flexible units is high, these units exercise market power to a greater extent and prices rise.

[11] It could be argued some very flexible power units, typically hydropower units, also feature low marginal costs. However these generators, as they are the most flexible, can choose to sell their production at any time-horizon. It is likely they will sell their production in earlier markets if prices are higher in these higher markets.

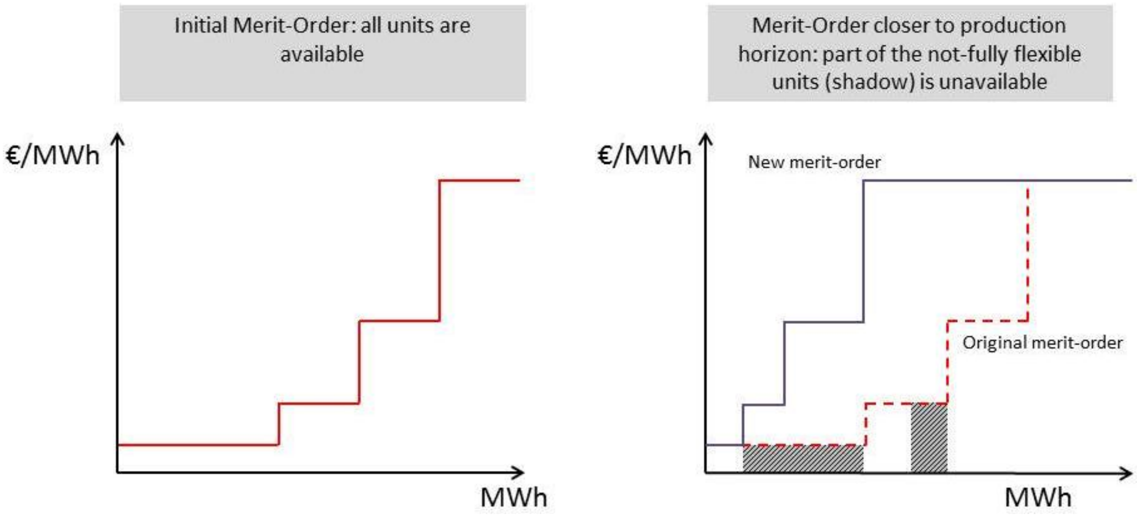

Figure 2: Evolution of the economic merit-order due to limited flexibility

3.2. Model implementation

Wind player behaviour

At time t_0, the wind player plans to generate a wind energy quantity w_0 at time t_n using the best available forecast.

The wind player is then given the possibility to adjust its position at a set of gates determined by market rules. Among the eligible gates, the player will choose to participate (adopt an active strategy) in *n-1* gates at times t_i , where $i \in [\![1, n-1]\!]$. This player is therefore taking part in *n+1* gates at times t_i , where $i \in [\![0, n]\!]$: t_0 is the day-ahead market gate closure time, t_n is the production horizon when electricity must be generated. For instance, in figure 2 the player decides to participate in IM at gates H-24, H-12, H-4 and H-2 and the t_i are then t_1 = H-24, t_2 = H-12, t_3 = H-4, t_4 = H-2 and t_5= H.

At time t_i, this player will then use the updated production forecast w_i. The player will cover the quantity q_i = w_0 - w_i buying energy from thermal generators. q_i is hereby defined as the net demand at time t_i. This player following the active strategy at time t_i and t_{i-1} will then buy the quantity $q_i - q_{i-1}$ at time t_i.

At the final time t_n, the wind player will cover the net demand q_n and pay the corresponding imbalance costs. A player having adopted the active strategy in gate t_{n-1} will be charged the costs corresponding to the remaining energy quantity $q_n - q_{n-1}$. By opposition, a player having adopted the passive strategy will be charged the costs corresponding to the energy quantity $q_n - q_0$.

The quantities w_i , $i \in [\![0, n]\!]$ are random variables whose behaviour depends on the wind farms characteristics and the wind nature itself. In order to make calculations simpler, we define the variable X_i representing the wind production forecast error at time t_i as a share of the realised wind production.

$$X_i = \frac{w_n - w_i}{w_n}$$

The resulting random variable X_i has an expected value $E(X_i) = 0$ and a variance σ_i^2. We suppose X_i and w_n are independent:

$$\forall\ i\ \in [\![0, n]\!], \qquad Cov(X_i, w_n) = 0$$

This simplification is made under the assumption that the forecast error X_i expressed as a share of the realised wind production is not correlated to the realised wind production w_n. In other words, there is no systematic relationship between wind power generation and wind power prediction accuracy[12]. Moreover $E(X_i) = 0$ indicates there is no systematic underestimation or overestimation at a given time. This is a very reasonable assumption as a forecasting tool presenting such a bias would be adjusted.

<u>Prices formation</u>

In our model wind power producers interact with thermal generators to buy the extra energy they need or to sell surplus energy. Demand-side is not considered as we suppose the balancing needs driven by the consumption-forecast error will be insignificant in a power system featuring high penetration by

[12] An example of empirical study analysing this property of wind power forecasts can be found in section 6 of Lange (2003).

intermittent RES[13]. The available thermal generators obey at time t_0 to the following aggregated inverse supply function. For a net demand q, the corresponding price $\bar{p}(q)$ is:

$$\bar{p}(q) = a + b.q$$

The price function is therefore linear and parameters a and b are inputs that depend on the power system properties. The variable b will be higher when the range of marginal costs of the different generation units will be higher.

The evolution of costs of dealing with imbalances will play a significant part in the trade-off wind generators are to face. To take flexibility into account in our model we introduce a "penalty function" $\varphi(t)$. We assume the value of the penalty function $\varphi(t)$ increases with time *t*: the extra cost of trading later is higher closer to real time.

We suppose a producer who committed at time t_{i-1} to buy the quantity q_{i-1} and trading the quantity $q_i - q_{i-1}$ at time t_i will pay a price $p(q_{i-1}, q_i, t_i)$. The resulting price function obeys to the following equation graphically illustrated in Figure 3 :

$$p(q_{i-1}, q_i, t_i) = \bar{p}\left(q_{i-1} + \left(1 + \varphi(t_i)\right) \times (q_i - q_{i-1})\right)$$

$$p(q_{i-1}, q_i, t_i) = \bar{p}(q_i) + b \times \varphi(t_i) \times (q_i - q_{i-1})$$

In case the system is not perfectly flexible (i.e. $\exists\, t \setminus \varphi(t) > 0$) the same quantity of electricity bought later by wind generators (when generation by thermal units is higher) will be more costly, while electricity sold later by wind generators (when generation by thermal units is lower) will lead to lower profits.

[13] While outages of thermal units will still be relevant for the network security we considered that due to the low frequency of occurrence they could be neglected in our financial analysis.

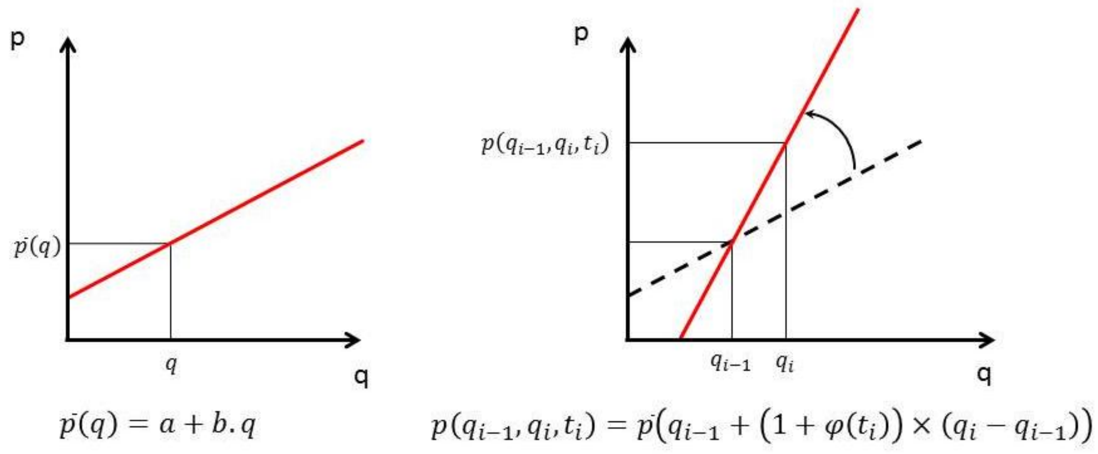

Figure 3: Evolution of the inverse supply function in our model

It is important to point out that representing the classical stepwise merit-order curve by a linear merit-order curve is a quite restrictive assumption. For a given time, in a real electricity market, start-up costs and additional non-convexities might challenge this hypothesis. However the scope of this article is to provide insights of phenomena taking place into IM, focusing on a single production hour. In this context, we considered that neglecting non-convexities constituted a reasonable assumption. The same argument also applies to the approximation by the supply function at different times t_i .

Picking the best strategy

A wind power producer having chosen to participate in IM at times t_i and t_{i-1} will trade the quantity $q_i - q_{i-1}$ at time t_i and pay a price $p(q_{i-1}, q_i, t_i)$. The total cost C_{IM} for a participants being active at times $t_i$where $i \in [\![1, n-1]\!]$ will therefore be the sum of these transactions[14]:

$$C_{IM}(q_0, \dots, q_n, t_1, \dots, t_n) = \sum_{i=1}^{n} [p(q_{i-1}, q_i, t_i) \times (q_i - q_{i-1})]$$

By opposition a producer staying completely out of the intraday market (what we defined as the passive strategy) will only buy the initial amount of energy at t_0 and pay the imbalance costs corresponding to quantity $q_n - q_0$ at time t_n. The total cost C_{NI} will then be:

$$C_{NI}(q_0, q_n, t_n) = p(q_0, q_n, t_n) \times (q_n - q_0)$$

[14] We consider that transaction costs are not significant and can be neglected in this study.

The player considered will be risk-neutral in our analysis. In order to compare the efficiency of these two strategies, the chosen *active strategy* and the *passive strategy*, we will have a look at the expected value of the difference between these two total costs $\Delta(t_1, \dots, t_n)$.

$$\Delta(t_1, \dots, t_n) = \bar{E}\big(C_{IM}(q_0, \dots, q_n, t_1, \dots, t_n) - C_{NI}(q_0, q_n, t_n)\big)$$

We will then compare the case of a player only active at times $t_1, \dots t_j, t_{j+1} \ \dots, t_n$ with the case of the player in addition active at time t_k with $t_j \leq t_k \leq t_{j+1}$. We will study the sign of $\Delta\big(t_1, \dots, t_j, t_k, t_{j+1}, \dots, t_n\big) - \Delta\big(t_1, \dots, t_j, t_{j+1}, \dots, t_n\big)$ to determine whether it is worth or not being active at time t_k in addition to $\mathrm{t_1, \dots, t_j, t_{j+1}, \dots, t_n}$.

4. Analytical results

4.1. General case

To express more precisely the value of $\Delta(t_1, \dots, t_n)$ it is necessary to introduce the correlation coefficient $r_{j,k}$ between X_j and X_k defined as: $r_{j,k} = \frac{Cov(X_j, X_k)}{\sigma_j \sigma_k}$

It is then possible to show the following result (see Appendix for demonstration):

$$\Delta(\ t_1, \dots, t_n) = b \times E(w_n^2) \times \left[\sum_{i=1}^{n} A_i + \sum_{i=1}^{n} B_i \ - C\right]$$

$A_i = {\sigma_i}^2 - r_{i-1,i}\sigma_i\sigma_{i-1}$

$B_i = \varphi(t_i) \times \big({\sigma_i}^2 + {\sigma_{i-1}}^2 - 2r_{i-1,i}\sigma_{i-1}\sigma_i\big)$

$C = {\sigma_0}^2 \times \varphi(t_n)$

This result can deliver a few insights. First of all, the costs of picking the wrong strategy (whether it is to play or not at a given time in intraday markets) will be proportional to both the slope of the supply curve b and the expected value of the square of wind power production $E(w_n^2)$. It is important to point out that $E(w_n^2)$ is higher when the average production is higher but also when the variability of the production is higher[15]. In a system where wind production is steadier, for example because the wind is itself more steady or because wind farms are more dispersed, the errors will also be less important. In a system where the marginal costs of thermal plants, flexible or not, are roughly the same, it will matter less which ones are called to generate.

Finally, the relevance of trading into these gates will be the result of a trade-off between the different members of this equation. The B_i terms are always positive and represent the "flexibility penalty" of buying energy latter in intraday markets when the generator adopts the active strategy. The term $-C$ is always negative and represents the same penalty paid in case the wind generator adopts a passive strategy. The value of the A_i term depends on the system characteristics and can be either positive or negative. If correlation $r_{i-1,i}$ between X_{i-1} and X_i is poor then losses resulting from oscillating predictions will be high and it might not be worth trading in intraday markets.

4.2. Results in a simple case with one gate closure in the intraday market

In a recent study of the Spanish electricity market, Furió et al. (2009) estimated that about two thirds of exchanges realised in the IM take place during the last possible platform. It means players use only one gate of the IM for a given hour. It is therefore interesting, in addition to being a good educational example, to study the case when the player is deciding whether to adjust its position (or not) at a single gate between the day-ahead electricity market and the generation time.

[15] Indeed $E(w_n^2) = \left(E(w_n)\right)^2 + Var(w_n)$

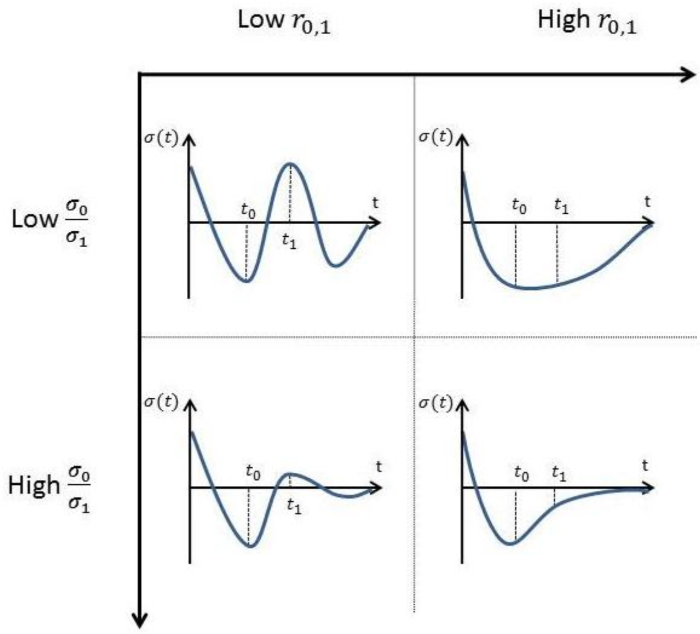

Figure 4: Examples of typical forecasts for given sets of parameters

Our approach consists in identifying for a given flexibility which forecasting abilities will lead to an active use of the additional gate. We introduce the ratio $\theta_{j,k}$:

$$\forall j < \ k \ \in [\![0, n-1]\!]^2, \qquad \theta_{j,k} = \frac{\sigma_j}{\sigma_k} \times r_{j,k}$$

$\theta_{j,k}$ is made of two components: $\frac{\sigma_j}{\sigma_k}$ indicates how much information is gained between t_j and t_k while $r_{j,k}$ is a measure of the correlation between these two pieces of information. An illustration with two steps is provided in Figure 4.

In our simple case when $n = 2$ we are able to identify two cases.

<u>**Lemma 1.1 (see demonstration in annex):**</u>

$n = 2$

For a player being given the possibility to trade at time t_1:

$\boldsymbol{\theta_{0,1} \geq 1} \Rightarrow \forall\, \varphi_{(t_1)}, \forall \varphi_{(t_2)}, \Delta(t_1, t_2) \leq 0$ *: it will be beneficial to adopt an active strategy at time t_1.*

Lemma 1.2 (see demonstration in annex):

$n = 2$

For a player being given the possibility to trade at time t_1:

$\boldsymbol{\theta_{0,1} \leq 1} => \exists\, \bar{\varphi} \,/\, \varphi_{(t_2)} \leq \bar{\varphi} => \forall\, \varphi_{(t_1)}, \Delta(t_1, t_2) \geq 0$ *: it will not be beneficial to adopt an active approach at time t_1.*

It is possible to go beyond these mathematical results and explore their meanings. In case $\frac{\sigma_0}{\sigma_1}$ is low, there is little interest in trading at t_1 since the forecast is not much more accurate. In case $r_{0,1}$ is low, there is little interest in trading at t_1 as there are higher risks of spoiling energy due to oscillating prediction errors. That's why $\theta_{0,1}$ is a key parameter.

From lemma 1.1, it is interesting for the producer to anticipate imbalances at t_1 if the forecast error evolution is good enough.

From Lemma 1.2, if the anticipation is not really helpful, i.e. $\theta_{0,1}$ is low, then it can be interesting or not to anticipate imbalances. If imbalances are never very expensive it is not worth taking the risk of a wrong anticipation.

4.3. Interest of trading at a given gate closure in the general case

Most intraday markets feature several gates (six in Spain) or allow continuous trading. Therefore we will have a look in this section at a general case when a participant is adjusting its position in $n-1$ gates in the IM at times t_i where $i \in [\![1, n-1]\!]$. We study the effects of being active at one more gate at time t_k and identify a set of criteria that will favour or discriminate against an active approach at this gate. By extension it is then possible to determine in which case a continuous market will be fully used by participants when n tends to infinity.

Lemma 2.1 (see demonstration in annex):

For a player adopting an active strategy in IM at gate closure times $t_1, \ldots t_j, t_{j+1}, \ldots, t_n$ being given the possibility to trade at time t_k with $t_j \leq t_k \leq t_{j+1}$:

$\begin{cases}\theta_{k,j+1} \geq \theta_{j,j+1} \\ \theta_{j,k} \geq 1\end{cases} \Rightarrow \Delta(t_1, \dots, t_j, t_k, t_{j+1}, \dots, t_n) \leq \Delta(t_1, \dots, t_j, t_{j+1}, \dots, t_n)$ *: it will be beneficial to adopt an active strategy at* t_k.

Lemma 2.2 (see demonstration in annex):

For a player adopting an active strategy in IM at gate closure times $t_1, \dots t_j, \dots, t_n$ *being given the possibility to trade at time* t_k *with* $t_j \leq t_k \leq t_{j+1}$ *:*

$$\begin{cases}\theta_{k,j+1} \leq \theta_{j,j+1} \\ \theta_{j,k} \leq 1\end{cases}$$

$\Rightarrow \exists\, \bar{\varphi} / \varphi_{(t_{j+1})} \leq \bar{\varphi} \Rightarrow \forall\, \varphi_{(t_k)}, \Delta(t_1, \dots, t_j, t_k, t_{j+1}, \dots, t_n) \geq \Delta(t_1, \dots, t_j, t_{j+1}, \dots, t_n)$ *: it will not be beneficial to adopt an active approach at* $t = t_k$.

We can deduce from lemma 2.2 that for a given flexibility of the power system and a specific forecast error evolution the active strategy might be more costly than the passive one. This result is coherent with the results obtained by Maupas (2008), De Vos et al. (2011) and Usaola and Angarita (2007).

5. Results interpretation

5.1. Liquidity in intraday markets

Conclusion 1: Low liquidity in intraday markets will be unavoidable for a given set of technical parameters.

A first insight we can get from our analysis is that poor liquidity in intraday markets may result from a rational behaviour of the participants. Our results indeed indicate that the poor liquidity of intraday markets could be explained by the poor information players have to deal with. Lemma 2.2 shows oscillating predictions can deter the players from trading in the IM provided it is not too expensive to procure energy in the balancing markets. This is an intuition already exposed by some of the authors mentioned in the section 2 of this article, but our results enlighten the key role played by the factor $\theta_{j,k}$. When the value of this parameter is low, it means the gain of information when getting closer to real-time is not sufficient to compensate the oscillating nature of wind forecasts. Participants acting rationally will then choose not to adjust their positions between day-ahead markets and real-time. Intraday markets will not be used by participants because they do not meet the needs of the participants.

Conclusion 2: In some cases, compelling players to trade into intraday markets will generate additional costs.

As long as conditions remain unsuitable, it will not be possible to increase both efficiency and liquidity by changing rules. Compelling wind power generators to trade in the intraday markets will mechanically lead to a more liquid intraday market, but these obligations can potentially result in higher total balancing costs. Higher volumes should not be the objective of regulators. The volume of exchanges in the intraday markets will spontaneously rise (or decrease) following a higher penetration of renewables or technological changes. A prerequisite is obviously that the intraday markets must be in place in the power system, even if they are not used by most participants. If the forecasting tools become good enough, producers will then apply voluntarily what we defined as the *active strategy*, in order to minimise their costs, as shown in lemma 2.1.

Similarly, setting penalties in real-time markets to incentivise participants to balance ex-ante their positions will lead to a higher participation in intraday markets, as in practice the extra cost $\varphi_{(t_n)}$ of trading in real-time will increase. However the actual costs of generating electricity will not be transformed by such financial penalties and these additional adjustments will not result in a higher efficiency. Increased participation in intraday markets will then be a form of hedge against imbalances with negative consequences similar to the ones described in Vandezande et al. (2010).

5.2. Trade-offs between continuous trading and discrete auctions

As mentioned in the introduction, there are two main options available to design intraday markets: continuous markets and discrete auctions (Barquin et al., 2011). In a continuous market, bids are matched one by one as soon as they match (i.e. when the bid price is higher than the offer price). The main alternative consists in a set of discrete auctions.

Conclusion 3: Setting discrete auctions in intraday markets may lead to inefficiencies due to lost trading opportunities.

By opposition to continuous markets, discrete auctions restrict trading to a set of pre-established times. Yet we know from our analysis that the strategy of a player will differ at different times. Depending on the wind forecast properties, a player might for instance be willing to trade at 10 a.m. but not at 9 a.m. or 11a.m. In a continuous market, players can use the experience they acquired day after day, and they will then be able to optimise their behaviour and trade when it is the most interesting for them. In a discrete market players will not be given such freedom: if conditions are not suitable (i.e. if the gates are set at times that do not fit this player) players will not trade, as shown by lemma 2.2.

That's why we argue restricting trading at imposed gates (as it is the case in an IM featuring discrete auctions) may lead to inefficiencies, additional costs, and lost trading opportunities. This result shall temper assumptions that discrete auctions will lead to increased trade in IM[16]. Obviously there are other sources of inefficiencies in continuous markets related to their inner fundamental properties: as trades are made on a first-come first-served basis in a continuous market, some trades that would not have taken place in a discrete market might take place, and the resulting prices will be less transparent. However, the decision to put into place continuous or discrete intraday markets should take into account the advantages of continuous markets that we described in addition to these drawbacks.

It could be argued that the gate-closure times could be set in a way to reflect players' preferences, which would only be theoretically possible in the case of a single balancing responsible party. Gates should in this case be set after analysing wind forecast evolutions and should be regularly updated as forecasting technologies and the generation park evolve. Such a painful administrative process could

[16] The case for discrete auctions is often illustrated by the relatively high liquidity in the Spanish intraday markets. Yet it is important to take into account the fact that in the Spanish electricity market, portfolio bidding is not allowed. Therefore, as underlined by Pérez Arriaga (2005), a significant share of the volumes exchanged in the intraday markets is due to internal re-allocation by participants of the dispatch resulting from the daily market. It is not the case in most other European electricity markets where portfolio bidding is implemented. Therefore the case of the Spanish IM should be exploited carefully.

be avoided by putting into place continuous markets. The losses would then offset the potential benefits from more efficient allocation in markets featuring discrete auctions.

6. Conclusion

In this paper, we assessed the different strategies that could be employed in intraday markets by parties responsible for managing wind forecast error. Participants trading in intraday markets face a trade-off: being exposed to imbalance charges or adjusting positions in the intraday market when some relevant information is still missing. Therefore we developed a simple analytical model allowing us to take into account both the system flexibility (as the lower the flexibility, the higher imbalance charges) and the nature of the wind forecast evolution (as it determines the information available to participants).

While discussions about optimal gate-closures usually focused on the average forecast error and the system flexibility when getting closer to real-time we demonstrated that correlation between forecast errors at different times should be taken into account. We were able to identify the parameter $\theta_{j,k}$ reflecting both the oscillating nature of wind forecasts and the level of information gained when getting closer to real-time. We showed this parameter plays a key-role in determining the participants' strategies.

Our analytical results underlined the fact that oscillating predictions could indeed explain the poor liquidity in IMs. In this case, a higher volume of exchanges in the intraday market should not be an objective *per se* as poor liquidity could simply reflect the fact taking part into these intraday markets will lead to higher costs: reducing total balancing costs should remain the main objective of regulated TSOs and regulators when establishing rules.

Our analysis also revealed it was unlikely a set of gates would please all participants. Players responsible for balancing wind low-predictability will achieve cost-optimisation spontaneously if they are given the opportunity to trade when they need it. We argue continuous markets provide participants with a sufficient degree of freedom to express their needs. While the liquidity remains low in continuous markets in place in Europe it should yet become naturally higher with an increasing share of renewables in the generation mix, as incentives to reduce costs should lead participants to optimise their participation in intraday markets. Lost opportunities resulting from setting discrete auctions might offset their benefits.

It must be pointed out that our model has been designed to provide general insights about the behaviour of wind players in intraday markets. As a consequence, rather strong assumptions have been employed, and the results obtained might therefore not be universally valid. Relaxing some of the assumptions described in section 3 should however not impact our results significantly: for instance start-up costs that we neglected tend to increase when getting closer to real-time and could be internalised in the supply function. In this paper, it has also been considered that players are risk-neutral. Risk-averse players might have stronger incentives to participate in IM (thus reducing their exposure to imbalances in real-time markets) but our results should not be qualitatively impacted when relaxing this assumption.

Another key-assumption we made is that wind power production is managed in a centralised way. While this assumption is close to reality in some power systems (such as Germany) it might not reflect the more complex situation in other power systems. This assumption is essential when considering that the system total imbalances are driven by the sign of our player imbalances: however our results will remain qualitatively true for any player whose imbalances are strongly (positively) correlated with the total system imbalances. This will in particular be the case if the main wind power producers own similar generation parks: a similar technology employed, in location with similar properties. A possible extension of our work could be to consider the interactions of several players managing only partly-correlated wind power sources.

Appendixes

A.1 Nomenclature

The following table contains a summary of the variables employed in this article.

Variable	Meaning
n	Number of gates after the day-ahead markets closure
t_0	Day-ahead market gate closure time
t_n	Production horizon
$t_i, i \in [\![1, n-1]\!]$	Closure time of the i^{th} gate of the intraday market
W	Total wind installed capacity
$w_i,\ i \in [\![0, n-1]\!]$	Forecasted wind output at t_i for the production horizon t_n
w_n	Realised wind output at the production horizon t_n
$q_i,\ i \in [\![0, n]\!]$	Net demand associated to w_i
$X_i,\ i \in [\![0, n-1]\!]$	Forecast error at time t_i as a share of the realised output
$E(X_i)$	Expected value of X_i
$\sigma_i^2,\ i \in [\![0, n-1]\!]$	Variance of X_i
$r_{j,k}$	Correlation coefficient between X_j and X_k
$\theta_{j,k}$	Ratio representing the quality of the forecast evolution (see 4.2)
a	Constant parameter of the inversed supply-function at time t_0
b	Slope of the inversed supply-function at time t_0
$\bar{p}(q)$	Price associated to a net demand q when all units are available
$\varphi(t)$	Function representing the extra-cost when trading at time t
$p(q_{i-1}, q_i, t_i)$	Price associated to demand $q_i - q_{i-1}$ at time t_i
C_{IM}	Costs associated to an active strategy in intraday markets
C_{NI}	Costs associated to a passive strategy in intraday markets
Δ	Expected value of the difference between C_{IM} and C_{NI}

A.2: Expression of $\boldsymbol{\Delta_\beta(t_1, \dots, t_n)}$

$$\Delta(t_1, \dots, t_n) = E\left(\sum_{i=1}^{n}[p(q_{i-1}, q_i, t_i) \times (q_i - q_{i-1})] - p(q_0, q_n, t_n) \times (q_n - q_0)\right) \tag{1}$$

By definition,

$$\forall\, q_{i-1}, q_i, t_i, \qquad p(q_{i-1}, q_i, t_i) = \bar{p}\left(q_{i-1} + \big(1 + \varphi(t_i)\big) \times (q_i - q_{i-1})\right) \tag{2}$$

And

$$\forall\, q, \qquad \bar{p}(q) = a + b.q \tag{3}$$

Thus by developing (1) we obtain:

$$\begin{aligned} \Delta(t_1, \dots, t_n) = {} & b \times E\left(\textstyle\sum_{i=1}^{n}[q_i \times (q_i - q_{i-1})] + \sum_{i=1}^{n}[\varphi(t_i) \times (q_i - q_{i-1})^2]\right) \\ & -b \times E\left(q_n \times (q_n - q_0) + \varphi(t_n) \times (q_n - q_0)^2\right) \end{aligned} \tag{4}$$

We will then estimate each of the four members of this equation

$$\boldsymbol{E\big(q_i \times (q_i - q_{i-1})\big)} = E\left(\left(\frac{q_n}{w_n} + \frac{q_i - q_n}{w_n}\right) \times \left(\frac{q_i - q_n}{w_n} - \frac{q_{i-1} - q_n}{w_n}\right) \times w_n^2\right)$$

As, by definition, $q_n = w_0 - w_n$ and $q_i - q_n = (w_0 - w_i) - (w_0 - w_n) = w_n - w_i$

$$\boldsymbol{E\big(q_i \times (q_i - q_{i-1})\big)} = E\left(\left(\frac{w_0 - w_n}{w_n} + \frac{w_n - w_i}{w_n}\right) \times \left(\frac{w_n - w_i}{w_n} - \frac{w_n - w_{i-1}}{w_n}\right) \times w_n^2\right)$$

Thus following notations defined in 3.2 we obtain:

$$\boldsymbol{E\big(q_i \times (q_i - q_{i-1})\big)} = E\big((X_i - X_0) \times (X_i - X_{i-1}) \times w_n^2\big)$$

And as by assumption (see section 3.2) $\forall\, i \in [\![0, n]\!]$, $Cov(X_i, w_n) = 0$ and $E(X_i) = 0$

$$\boldsymbol{E}\big(\boldsymbol{q_i} \times (\boldsymbol{q_i} - \boldsymbol{q_{i-1}})\big) = E(w_n^2) \times E\big((X_i - X_0) \times (X_i - X_{i-1})\big)$$

Following notations defined in 3.2 we obtain $\forall\ (j,k)\ \in [\![0,n]\!]^2$:

$$E(X_j X_k) = Cov(X_j, X_k) + E(X_j) \times E(X_k)$$

And, as $E(X_j) = 0$ and by definition $Cov(X_j, X_k) = r_{j,k}\sigma_j\sigma_k$

$$E(X_j X_k) = r_{j,k}\sigma_j\sigma_k$$

$$\boldsymbol{E}\big(\boldsymbol{q_i} \times (\boldsymbol{q_i} - \boldsymbol{q_{i-1}})\big) = E(w_n^2) \times \big({\sigma_i}^2 - r_{i-1,i}\sigma_i\sigma_{i-1} - r_{0,i}\sigma_0\sigma_i + r_{0,i-1}\sigma_0\sigma_{i-1}\big) \qquad (4.1)$$

And by a similar process:

$$\boldsymbol{E}\big((\boldsymbol{q_i} - \boldsymbol{q_{i-1}})^{\boldsymbol{2}}\big) = E(w_n^2) \times \big({\sigma_i}^2 + {\sigma_{i-1}}^2 - 2 \times r_{i-1,i}\sigma_i\sigma_{i-1}\big) \qquad (4.2)$$

$$\boldsymbol{E}\big(\boldsymbol{q_n} \times (\boldsymbol{q_n} - \boldsymbol{q_0})\big) = E(w_n^2) \times r_{0,0}\sigma_0\sigma_0 \qquad (4.3)$$

$$\boldsymbol{E}\big((\boldsymbol{q_n} - \boldsymbol{q_0})^{\boldsymbol{2}}\big) = E(w_n^2) \times {\sigma_0}^2 \qquad (4.4)$$

Moreover:

$\sum_{i=1}^{n}\big(-r_{0,i}\sigma_0\sigma_i + r_{0,i-1}\sigma_0\sigma_{i-1}\big) - r_{0,0}\sigma_0\sigma_0 = -\, r_{0,n}\sigma_0\sigma_n = 0$, as $\sigma_n = 0$

We can therefore write (4) as

$$\Delta(\,t_1, \dots, t_n) = b \times E(w_n^2) \times \left[\sum_{i=1}^{n} A_i + \sum_{i=1}^{n} B_i\ - C\right] \qquad (5)$$

Where:

$A_i = {\sigma_i}^2 - r_{i-1,i}\sigma_i\sigma_{i-1}$

$B_i = \varphi(t_i) \times \left(\sigma_i{}^2 + \sigma_{i-1}{}^2 - 2r_{i-1,i}\sigma_{i-1}\sigma_i\right)$

$C = \sigma_0{}^2 \times \varphi(t_n)$

A.3: Proof of lemma 1.1:

We apply equation (5) in the special case when $n = 2$

$$\frac{\Delta(\, t_1, t_2)}{b \times E(w_n^2))} = \sigma_1{}^2 - r_{0,1}\sigma_0\sigma_1 + \varphi(t_1) \times \left(\sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1\right) + \varphi(t_2) \times \sigma_1{}^2 - \varphi(t_2) \times \sigma_0{}^2 \qquad (6)$$

As $|r_{0,1}| \le 1,\ \sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1 \ge \sigma_1{}^2 + \sigma_0{}^2 - 2\sigma_0\sigma_1 \ge (\sigma_0 - \sigma_1)^2 \ge 0$

Hence $\Delta\left(\, t_1, t_2\right) \le 0 <=> \varphi_\beta(t_1) \le\ \varphi\ (t_2) \times \frac{(\sigma_0{}^2 - \sigma_1{}^2)}{\sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1} - \ \frac{\sigma_1{}^2 - r_{0,1}\sigma_0\sigma_1}{\sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1}$ (7)

We assume that $\sigma_0{}^2 \ge \sigma_1{}^2$: the uncertainty increases with the prediction horizon.

If we assume $\theta_{0,1} \ge 1\ <=> \sigma_0 \times r_{0,1} \ge\ \sigma_1$ we obtain the following results:

$$\frac{\sigma_1{}^2 - r_{0,1}\sigma_0\sigma_1}{\sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1} \le 0 \qquad (7.1)$$

And as $\sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1 \ = \sigma_0{}^2 - \sigma_1{}^2 + 2\left(\sigma_1{}^2 - r_{0,1}\sigma_0\sigma_1\right) \le \sigma_0{}^2 - \sigma_1{}^2$

$$\frac{(\sigma_0{}^2 - \sigma_1{}^2)}{\sigma_1{}^2 + \sigma_0{}^2 - 2r_{0,1}\sigma_0\sigma_1} \ge\ 1 \qquad (7.2)$$

We also know that $\varphi(t_1) \le \varphi(t_2)$ as the flexibility penalty $\varphi(t)$ increases with t

$$=> \forall\, \gamma_1 \ge 1, \forall\, \gamma_2 \le 0, \qquad \varphi(t_1) \le \gamma_1 \times \varphi(t_2) - \gamma_2 \qquad (8)$$

Using (7.1), (7.2) and (8) we can show that the following equation is verified

$$\varphi(t_1) \leq \varphi(t_2) \times \frac{({\sigma_0}^2 - {\sigma_1}^2)}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} - \frac{{\sigma_1}^2 - r_{0,1}\sigma_0\sigma_1}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} \tag{9}$$

And according to (7) and (9), $\Delta\left(t_1, t_2\right) \leq 0$

A.4: Proof of lemma 1.2:

We assume $\theta_{0,1} \leq 1 <=> \sigma_0 \times r_{0,1} \leq \sigma_1$

By analogy to the proofs of (7.1) and (7.2), we can show:

$$0 \leq \frac{{\sigma_1}^2 - r_{0,1}\sigma_0\sigma_1}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} \leq 1 \tag{10.1}$$

$$0 \leq \frac{({\sigma_0}^2 - {\sigma_1}^2)}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} \leq 1 \tag{10.2}$$

And therefore:

$$\exists\, \bar{\varphi} \,/\, \forall\, \varphi_{(t_2)} \leq \bar{\varphi} => \varphi(t_2) \times \frac{({\sigma_0}^2 - {\sigma_1}^2)}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} - \frac{{\sigma_1}^2 - r_{0,1}\sigma_0\sigma_1}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} \leq 0 \tag{11}$$

As by definition $\varphi(t_1) \geq 0$

$$\exists\, \bar{\varphi} \,/\, \varphi_{(t_2)} \leq \bar{\varphi} => \varphi(t_1) \geq \varphi(t_2) \times \frac{({\sigma_0}^2 - {\sigma_1}^2)}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} - \frac{{\sigma_1}^2 - r_{0,1}\sigma_0\sigma_1}{{\sigma_1}^2 + {\sigma_0}^2 - 2r_{0,1}\sigma_0\sigma_1} \tag{12}$$

Using (7), $\exists\, \bar{\varphi} \,/\, \varphi_{(t_2)} \leq \bar{\varphi} => \forall\, \varphi_{(t_1)}, \Delta(t_1, t_2) \geq 0$

A.5: Proof of lemma 2.1:

A player adopting an active strategy in IM at gate closure times $t_1, \ldots t_j, t_{j+1}, \ldots, t_n$ is being given the possibility to trade at time t_k with $t_j \leq t_k \leq t_{j+1}$.

We make two assumptions.

<u>*Assumption 1:*</u> $\theta_{k,j+1} \geq \theta_{j,j+1}$

<u>*Assumption 2:*</u> $\theta_{j,k} \geq 1$

By definition it will be beneficial to adopt an active strategy at t_k if and if only:

$$\Delta(t_1, \dots, t_j, t_k, t_{j+1}, \dots, t_n) \leq \Delta(t_1, \dots, t_j, t_{j+1}, \dots, t_n) \tag{13}$$

Most of the terms are present on each side and by developing and simplifying (13) is equivalent to

$$\begin{aligned} 0 \geq \sigma_k^2 - r_{j,k}\sigma_j\sigma_k - r_{k,j+1}\sigma_k\sigma_{j+1} + r_{j,j+1}\sigma_j\sigma_{j+1} + \varphi(t_k) \times \left({\sigma_j}^2 + {\sigma_k}^2 - 2r_{j,k}\sigma_j\sigma_k\right) \\ + \varphi(t_{j+1}) \times \left({\sigma_k}^2 - {\sigma_j}^2 - 2\, r_{k,j+1}\sigma_k\sigma_{j+1} + 2r_{j,j+1}\sigma_j\sigma_{j+1}\right) \end{aligned} \tag{14}$$

Under assumption 1, $\theta_{k,j+1} \geq \theta_{j,j+1}$ and therefore

$$r_{k,j+1}\sigma_k\sigma_{j+1} \geq r_{j,j+1}\sigma_j\sigma_{j+1} \tag{15.1}$$

In addition we know that we assumed greater uncertainty further away from the production horizon:

$${\sigma_j}^2 \geq {\sigma_k}^2 \tag{15.2}$$

Using (15.1) and (15.2)

$${\sigma_j}^2 - {\sigma_k}^2 + 2\, r_{k,j+1}\sigma_k\sigma_{j+1} - 2r_{j,j+1}\sigma_j\sigma_{j+1} \geq 0 \tag{16}$$

And according to (16) it is possible to rewrite (14) as:

$$\varphi(t_{j+1}) \geq \varphi(t_k) \times \frac{{\sigma_j}^2 + {\sigma_k}^2 - 2r_{j,k}\sigma_j\sigma_k}{{\sigma_j}^2 - {\sigma_k}^2 + 2\, r_{k,j+1}\sigma_k\sigma_{j+1} - 2r_{j,j+1}\sigma_j\sigma_{j+1}} + \frac{\sigma_k^2 - r_{j,k}\sigma_j\sigma_k + r_{j,j+1}\sigma_j\sigma_{j+1} - r_{k,j+1}\sigma_k\sigma_{j+1}}{{\sigma_j}^2 - {\sigma_k}^2 + 2\, r_{k,j+1}\sigma_k\sigma_{j+1} - 2r_{j,j+1}\sigma_j\sigma_{j+1}} \tag{17}$$

We know have to show that inequality (17) is true to ensure that inequality (13) is true.

Under assumption 2: $\theta_{j,k} \geq 1$ and $\sigma_k^2 \leq r_{j,k}\sigma_j\sigma_k$

Under assumption 1: $\theta_{k,j+1} \geq \theta_{j,j+1}$ and $\sigma_k^2 r_{j,j+1}\sigma_j\sigma_{j+1} \leq r_{k,j+1}\sigma_k\sigma_{j+1}$

$$\frac{\sigma_k^2 - r_{j,k}\sigma_j\sigma_k + r_{j,j+1}\sigma_j\sigma_{j+1} - r_{k,j+1}\sigma_k\sigma_{j+1}}{\sigma_j{}^2 - \sigma_k{}^2 + 2\, r_{k,j+1}\sigma_k\sigma_{j+1} - 2r_{j,j+1}\sigma_j\sigma_{j+1}} \leq 0 \qquad (18.1)$$

Besides $r_{j,j+1}\sigma_j\sigma_{j+1} \leq r_{k,j+1}\sigma_k\sigma_{j+1}$

$=> \frac{\sigma_j{}^2+\sigma_k{}^2-2r_{j,k}\sigma_j\sigma_k}{\sigma_j{}^2-\sigma_k{}^2+2\, r_{k,j+1}\sigma_k\sigma_{j+1}-2r_{j,j+1}\sigma_j\sigma_{j+1}} \leq \frac{\sigma_j{}^2-\sigma_k{}^2+2\sigma_k{}^2-2r_{j,k}\sigma_j\sigma_k}{\sigma_j{}^2-\sigma_k{}^2}$

$=> \frac{\sigma_j{}^2+\sigma_k{}^2-2r_{j,k}\sigma_j\sigma_k}{\sigma_j{}^2-\sigma_k{}^2+2\, r_{k,j+1}\sigma_k\sigma_{j+1}-2r_{j,j+1}\sigma_j\sigma_{j+1}} \leq 1 + 2 \times \frac{\sigma_k{}^2-r_{j,k}\sigma_j\sigma_k}{\sigma_j{}^2-\sigma_k{}^2}$

$$As\ \sigma_k^2 \leq r_{j,k}\sigma_j\sigma_k,\ \frac{\sigma_j{}^2+\sigma_k{}^2-2r_{j,k}\sigma_j\sigma_k}{\sigma_j{}^2-\sigma_k{}^2+2\, r_{k,j+1}\sigma_k\sigma_{j+1}-2r_{j,j+1}\sigma_j\sigma_{j+1}} \leq 1 \qquad (18.2)$$

We also know that by definition $\varphi(t_{j+1}) \geq \varphi(t_k)$ as the flexibility penalty $\varphi(t)$ increases with t. Hence using (18.1) and (18.2):

$$\varphi(t_{j+1}) \geq \varphi(t_k) \times \frac{\sigma_j{}^2+\sigma_k{}^2-2r_{j,k}\sigma_j\sigma_k}{\sigma_j{}^2-\sigma_k{}^2+2\, r_{k,j+1}\sigma_k\sigma_{j+1}-2r_{j,j+1}\sigma_j\sigma_{j+1}} + \frac{\sigma_k^2-r_{j,k}\sigma_j\sigma_k+r_{j,j+1}\sigma_j\sigma_{j+1}-r_{k,j+1}\sigma_k\sigma_{j+1}}{\sigma_j{}^2-\sigma_k{}^2+2\, r_{k,j+1}\sigma_k\sigma_{j+1}-2r_{j,j+1}\sigma_j\sigma_{j+1}} \qquad (19)$$

And (17) is verified, which is equivalent to $\Delta(t_1, \dots, t_j, t_k, t_{j+1}, \dots, t_n) \leq \Delta(t_1, \dots, t_j, t_{j+1}, \dots, t_n)$: it is beneficial to play the active strategy.

A.6: Proof of lemma 2.2:

Similar to 1.2 using equation (17).

References

Barquin, J., Rouco, L., Rivero, E., 2011. Current designs and expected evolutions of Day-ahead, Intra-day and balancing market/mechanisms in Europe, in: OPTIMATE (Ed.).
Besnier, D., 2009. Impact of the law on Renewable Energy ("EEG") on EPEX Spot German/Austrian Auction, EPEX SPOT INFO.
Borggrefe, Neuhoff, K., 2011. Balancing and intra-day market design: options for wind integration, in: Initiative, C.P. (Ed.), European Smart Power Market Project.
De Vos, K., De Rijcke, S., Driesen, J., Kyriazis, A., 2011. Value of Market Mechanisms Enabling Improved Wind Power Predictions: A Case Study of the Estinnes Wind Power Plant. status: published.
Furió, D., Lucia, J.J., Meneu, V., 2009. The Spanish Electricity Intraday Market: Prices and Liquidity Risk. Current Politics and Economics of Europe 20, 1-22.
Hiroux, C., Saguan, M., 2010. Large-scale wind power in European electricity markets: Time for revisiting support schemes and market designs? Energ Policy 38, 3135-3145.
IEA, 2012. The Impact of Wind power on European Natural Gas Markets.
Lange, M., 2003. Analysis of the uncertainty of wind power predictions. Department of Mathematics, 128.
Maupas, F., 2008. Analyse de l'impact économique de l'aléa éolien sur la gestion de l'équilibre d'un système électrique. Paris-XI.
Perez-Arriaga, I.J., 2012. Managing large scale penetration of intermittent renewables. MIT.
Stoft, S., 2002. Power system economics: designing markets for electricity. IEEE press.
Usaola, J., Angarita, J., 2007. Bidding wind energy under uncertainty. IEEE, pp. 754-759.
Vandezande, L., Meeus, L., Belmans, R., Saguan, M., Glachant, J.M., 2010. Well-functioning balancing markets: A prerequisite for wind power integration. Energ Policy 38, 3146-3154.
Weber, C., 2010. Adequate intraday market design to enable the integration of wind energy into the European power systems. Energ Policy 38, 3155-3163.